Introduction

Why do we want daily indexes?
What will we do in this study?

US space markets. NOI per square foot (per unit for apartment, right axis.)

US asset markets. E/P ratios.
Connection with previous literature

Methodology

Data

Results - Estimates on λ

Results - Indexes

Fit

December 2020 forecast and the GFC
Introduction

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Results - Estimates on \( \lambda \)

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Fit

December 2020 forecast and the GFC

Why do we want daily indexes?

- In the US, commercial real estate is among the largest asset classes;
  - Total value is estimated to be $18 Trillion in 2018 (Zillow & NAREIT).
  - Total value of "investable" commercial real estate is estimated to be $5 Trillion, of which $1 Trillion is owned by REITs (Real Capital Analytics).
  - By comparison, single family housing is valued at $23 Trillion in 2019 (Zillow).

- Having daily indexes can therefore help banks, and policy makers to identify bubbles and crashes in a more timely manner.

- Also, roughly the same group of investors that fall under the NCREIF (National Council of Real Estate Investment Fiduciaries) family are mandated to provide daily valuations of their AUM.
  - These include pension funds and insurance funds that operate under a defined contribution plan, or 401(k).
  - Pension/Insurance funds have around $250 Billion in real estate AUM (own calculations), about half of that falls under defined contribution.
In this research we provide a new methodology that allows us to give daily valuations of **real estate portfolios**.

The only input we need is a daily change in Net-Operating-Income. The change in portfolio value due a changes in P/E ratios (plus something else, more on that later) comes from our model;

- The **monthly returns** of a repeat sales model (with underlying data from Real Capital Analytics), are modeled explicitly as a function of **daily returns** realized by REITs. More specifically, we use changes in stock values (so levered) of an aggregate of REITs provided to us by NAREIT.

- To reduce some of the “stock market noise” we apply a weekly moving average on our daily returns. However, the “purely” daily results, and even monthly and quarterly MA smoothed results, are readily available if someone is interested. (I left it out to conserve space.)

Specifically for this seminar we also prepared a 6 month forecast using our model.
Introduction
Why do we want daily indexes?
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US space markets. NOI per square foot (per unit for apartment, right axis.)

US asset markets. E/P ratios.
Connection with previous literature

Methodology

Data

Results - Estimates on $\lambda$

Results - Indexes

Fit

December 2020 forecast and the GFC
Introduction
Why do we want daily indexes?
What will we do in this study?

US asset markets. E/P ratios.

US space markets. NOI per square foot (per unit for apartment, right axis.)

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Connection with previous literature

Methodology

Data

Results - Estimates on $\lambda$

Results - Indexes

Fit

December 2020 forecast and the GFC
Daily pricing for single family housing has already been researched, see;

- Bollerslev, Patton, and Wang, 2016, JAE.

However, this has not been the case for commercial real estate;

- Lack of transactions.
- Lack of characteristics.
- Large amount of heterogeneity.

Structural time series repeat sales models. See for example;

- Francke, 2010; Francke and van de Minne, 2017; van de Minne et al., forthcoming, JREFE.

Mixed frequency modelling and temporal aggregation. See for example;

- Ghysels, Sinko, and Valkanov, 2007, ER and Proietti, 2006, EJ

The relationship between public and private real estate. See for example;

- Barkham and Geltner, 1995; Geltner and Kluger, 1998, REE
Methodology

Repeat Sales Model (1/2)
Repeat Sales Model (2/2)
The State Equation
Mixed Frequency (1/2)
Mixed Frequency (2/2)

Data

Results - Estimates on $\lambda$
Results - Indexes

Fit

December 2020 forecast and the GFC
First of all, our private real estate data is a cross sectional dataset of real estate transactions.

To estimate our time series (capital gains), we use a repeat sales framework.

Normal hedonic Equation is given by;

\[
\ln P_{it} = \mu_t + X_{it}\beta + Z_{it}\alpha + \epsilon_{it},
\]

with;

- \( \mu \) being time dummies at time \( t \).
- \( X \) being the observed characteristics of property \( i \), with corresponding vector of coefficients \( \beta \).
- \( Z \) being the unobserved characteristics of property \( i \), with corresponding vector of coefficients \( \zeta \).
- The residuals are given by \( \epsilon \) which is assumed to be normally distributed; \( \epsilon \sim N(0, \sigma^2_\epsilon) \).
With repeat sales model, we replace all observed (and hopefully unobserved) characteristics with property level dummies;

$$\ln P_{it} = \mu_t + \delta_i + \epsilon_{it},$$ \hspace{1cm} (2) 

with $\delta_i$ being the property fixed effect.

Things become even more tractable when modelling the returns, as the property fixed effect cancels;

$$\ln P_{it} - \ln P_{is} = \mu_t - \mu_s + \epsilon_{it} - \epsilon_{is},$$ \hspace{1cm} (3) 

where $s$ is time of buy, and $t$ is time of sell.

Finally, we add the change in Net Operating Income (NOI) to our repeat sales model, as this is an important input (on a daily basis) to explain changes in prices. This gives us measurement Eq;

$$\ln P_{it} - \ln P_{is} = \mu_t - \mu_s + \omega(\ln NOI_{it} - \ln NOI_{is}) + \epsilon_{it} - \epsilon_{is}. \hspace{1cm} (4)$$
There are two issues with the repeat sales model for our application;

☐ Estimates of index can be “noisy” due to limited observations and heterogeneity.

☐ Our (private) real estate data is on a monthly frequency.

To kill two birds with one stone, we use a structural time series approach, in which we model our monthly returns as a function of higher frequency REIT prices.

For the state Eq. we therefore get;

$$\Delta \mu_t \sim \left( \sum_{j=0}^{l} \lambda_j \Delta \bar{x}_{mt-j}^{(k)} ; \sigma_{\mu} \right). \quad (5)$$

If $\tau$ is the higher frequency $k$ (say days), then the REIT returns ($\Delta x_{\tau}$) are observed at $\tau = (t - 1)m + 1, \ldots, tm$.

Note that there are two inputs; the frequency and the amount of lags.
As a quick example, say we estimate a monthly index ($\mu$).

The REIT returns are averages over a week, meaning that $m = 4$. (In other words there are always 4 weeks in 1 month.)

We take up two months worth of lags of REITS, or $4 \text{ weeks } \times 2 \text{ months} = 8$ coefficients in vector $\lambda$.

We get:

$$
\begin{bmatrix}
\Delta \mu_2 \\
\vdots \\
\Delta \mu_T
\end{bmatrix} =
\begin{bmatrix}
\Delta x_8 & \cdots & \Delta x_1 \\
\vdots & \ddots & \vdots \\
\Delta x_{4T} & \cdots & \Delta x_{4T-7}
\end{bmatrix}
\begin{bmatrix}
\lambda_1 \\
\vdots \\
\lambda_8
\end{bmatrix} +
\begin{bmatrix}
\eta_2 \\
\vdots \\
\eta_T
\end{bmatrix}.
$$

(6)

If $\mu_t$ would be simple univariate time series data, this model could be estimated by means of OLS.
We found the most interesting results (but not per se the best fit) if fix our lags, so they always add up to 1 years worth of lags.

This also means the we potentially have a lot of coefficients in vector $\lambda$.

The most widely used method in literature is the so-called Almon transformation, like quadratic or exponential Almon functions. However, we found that this did terribly in our application.

Instead, we use a random walk on the parameter:

$$\Delta \lambda_j \sim N(0, \sigma_\lambda),$$  \hspace{1cm} (7)

$$\lambda_1 \sim N(0, 1).$$  \hspace{1cm} (8)

We have experimented with different frequency for $\tau$.

The model is estimated using the No-U-Turn Sampler (which is proper MCMC, akin to HMC).

We denote our model the Mixed (K) Frequency repeat sales model (MKF).
We utilize two sources;

- **Real Capital Analytics (RCA).** Repeat sales transaction data of CRE between 2006 - 2020, including NOI.
- **NAREIT.** Composite index of stock prices of listed REITs, both all and for only apartments.

**RCA data for all properties:**

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>sd</th>
<th>1Qrt</th>
<th>3Qrt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>$ 45,619,870</td>
<td>$100,052,553</td>
<td>$7,050,209</td>
<td>$ 45,000,000</td>
</tr>
<tr>
<td>- return</td>
<td>0.172</td>
<td>0.380</td>
<td>-0.021</td>
<td>0.402</td>
</tr>
<tr>
<td>- return (YoY)</td>
<td>0.053</td>
<td>0.109</td>
<td>-0.004</td>
<td>0.102</td>
</tr>
<tr>
<td>NOI</td>
<td>$ 2,574,722</td>
<td>$ 5,055,628</td>
<td>$441,771</td>
<td>$ 2,677,553</td>
</tr>
<tr>
<td>- return</td>
<td>0.105</td>
<td>0.349</td>
<td>-0.085</td>
<td>0.288</td>
</tr>
<tr>
<td>- return (YoY)</td>
<td>0.032</td>
<td>0.106</td>
<td>-0.016</td>
<td>0.065</td>
</tr>
<tr>
<td>Year of sale</td>
<td>2013</td>
<td>4</td>
<td>2009</td>
<td>2016</td>
</tr>
</tbody>
</table>

**N**

10,482
We utilize two sources:

- **Real Capital Analytics (RCA).** Repeat sales transaction data of CRE between 2006 - 2020, including NOI.
- **NAREIT.** Composite index of stock prices of listed REITs, both all and for only apartments.

### RCA data for Apartments:

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>sd</th>
<th>1Qtr</th>
<th>3Qtr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>$22,891,264</td>
<td>$27,319,159</td>
<td>$5,400,000</td>
<td>$31,300,000</td>
</tr>
<tr>
<td>- return</td>
<td>0.319</td>
<td>0.338</td>
<td>0.134</td>
<td>0.507</td>
</tr>
<tr>
<td>- return (YoY)</td>
<td>0.083</td>
<td>0.100</td>
<td>0.026</td>
<td>0.131</td>
</tr>
<tr>
<td>NOI</td>
<td>$1,262,857</td>
<td>$1,382,152</td>
<td>$314,558</td>
<td>$1,801,016</td>
</tr>
<tr>
<td>- return</td>
<td>0.243</td>
<td>0.340</td>
<td>0.049</td>
<td>0.427</td>
</tr>
<tr>
<td>- return (YoY)</td>
<td>0.064</td>
<td>0.113</td>
<td>0.010</td>
<td>0.098</td>
</tr>
<tr>
<td>Year of sale</td>
<td>2013</td>
<td>4</td>
<td>2010</td>
<td>2017</td>
</tr>
</tbody>
</table>

N = 3,804
NAREIT indexes.

Results - Indexes

December 2020 forecast and the GFC
Results - Estimates on λ

Introduction

Methodology

Data

Results - Indexes

Fit

December 2020 forecast and the GFC

All property types

Apartment

Apartment with all types

NAREIT returns
All property types

Apartment
Apartment with all types
NAREIT returns

Results - Estimates on $\lambda$

December 2020 forecast and the GFC
Apartment

Introduction

Methodology

Data

Results - Estimates on $\lambda$

All property types

Apartment

Apartment with all types

NAREIT returns

Results - Indexes

Fit

December 2020 forecast and the GFC
Results - Indexes

Indexes - all types
Indexes - Apartment
Return Statistics and Estimates

Fit

December 2020 forecast and the GFC
Return Statistics and Estimates

<table>
<thead>
<tr>
<th></th>
<th>All types</th>
<th></th>
<th>Apartments</th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baseline</td>
<td>MKF</td>
<td>Baseline</td>
<td>MKF</td>
<td>MKF - Robust</td>
</tr>
<tr>
<td>mean (YoY)</td>
<td>0.007</td>
<td>0.005</td>
<td>0.020</td>
<td>0.020</td>
<td>0.020</td>
</tr>
<tr>
<td>sd</td>
<td>0.013</td>
<td>0.011</td>
<td>0.009</td>
<td>0.008</td>
<td>0.008</td>
</tr>
<tr>
<td>min</td>
<td>-0.035</td>
<td>-0.041</td>
<td>-0.022</td>
<td>-0.027</td>
<td>-0.026</td>
</tr>
<tr>
<td>max</td>
<td>0.027</td>
<td>0.027</td>
<td>0.027</td>
<td>0.018</td>
<td>0.020</td>
</tr>
</tbody>
</table>

Parameter Estimates

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>0.751</td>
<td>0.751</td>
<td>0.614</td>
<td>0.614</td>
<td>0.614</td>
</tr>
<tr>
<td>$\sigma_\epsilon$</td>
<td>0.227</td>
<td>0.228</td>
<td>0.182</td>
<td>0.182</td>
<td>0.182</td>
</tr>
<tr>
<td>$\sigma_\mu$</td>
<td>0.020</td>
<td>0.013</td>
<td>0.018</td>
<td>0.013</td>
<td>0.012</td>
</tr>
<tr>
<td>$\sigma_\lambda$</td>
<td>-</td>
<td>0.011</td>
<td>-</td>
<td>0.008</td>
<td>0.007</td>
</tr>
</tbody>
</table>
Fit

Revision Analysis
Index revisions (all types) of Baseline model - including forecast.
Index revisions (all types) of the MKF model - including forecast.
Revision of parameters (1/2); Net-Operating-Income (ω)
Revision of parameters (2/2); REIT returns (λ)
MAPE Statistics - all types.
MAPE Statistics - Apartments.
December 2020 forecast and the GFC
Measuring model fit of indexes is challenging, because standard fit statistics are all based on cross-sectional residuals.

In order to measure fit, we will do a revision analysis.

- First we run the model using data only up to Jan 2015.
- We subsequently estimate the model using only data up to Feb 2015, all the way to the end of our sample, which is Jun 2020.
- This gives us 65 different indexes.
- We subsequently compute the statistics on the following returns:
  1. We forecast 1 month out and compare the forecast error with the actual return.
  2. We compare the final period’s revisions. (i.e. the “embarrassing revision”.)
  3. We compute statistics on all revisions, not just the final period(s).
Index revisions (all types) of Baseline model - including forecast.

Revision Analysis
Index revisions (all types) of Baseline model - including forecast.
Index revisions (all types) of the MKF model - including forecast.
Revision of parameters (1/2); Net-Operating-Income ($\omega$)
Revision of parameters (2/2); REIT returns ($\lambda$)
MAPE Statistics - all types.
MAPE Statistics - Apartments.

December 2020 forecast and the GFC
Index revisions (all types) of the MKF model - including forecast.

Revision Analysis
Index revisions (all types) of Baseline model - including forecast.

Revision of parameters (1/2); Net-Operating-Income ($\omega$)
Revision of parameters (2/2); REIT returns ($\lambda$)
MAPE Statistics - all types.
MAPE Statistics - Apartments.
December 2020 forecast and the GFC.
Revision of parameters (2/2); REIT returns ($\lambda$)

Introduction
Methodology
Data
Results - Estimates on $\lambda$
Results - Indexes
Fit
Revision Analysis
Index revisions (all types) of Baseline model - including forecast.
Index revisions (all types) of the MKF model - including forecast.
Revision of parameters (1/2); Net-Operating-Income ($\omega$)
Revision of parameters (2/2); REIT returns ($\lambda$)
MAPE Statistics - all types.
MAPE Statistics - Apartments.
December 2020 forecast and the GFC.
MAPE Statistics - all types.

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>mape</th>
<th>sd</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Baseline</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>forecast</td>
<td>-0.00122</td>
<td>0.01072</td>
<td>0.01285</td>
<td>-0.02693</td>
<td>0.02751</td>
</tr>
<tr>
<td>final period</td>
<td>-0.00027</td>
<td>0.00396</td>
<td>0.00471</td>
<td>-0.01045</td>
<td>0.01070</td>
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<tr>
<td>all</td>
<td>-0.00001</td>
<td>0.00012</td>
<td>0.00015</td>
<td>-0.00037</td>
<td>0.00033</td>
</tr>
<tr>
<td><strong>MKF repeat sales model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>forecast</td>
<td>-0.00005</td>
<td>0.00525</td>
<td>0.00662</td>
<td>-0.01595</td>
<td>0.01892</td>
</tr>
<tr>
<td>final period</td>
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<td>0.00288</td>
<td>0.00363</td>
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<td>0.00775</td>
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<tr>
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<td>0.00000</td>
<td>0.00008</td>
<td>0.00010</td>
<td>-0.00024</td>
<td>0.00023</td>
</tr>
</tbody>
</table>

Revision Analysis
Index revisions (all types) of Baseline model - including forecast.
Index revisions (all types) of the MKF model - including forecast.
Revision of parameters (1/2); Net-Operating-Income ($\omega$)
Revision of parameters (2/2); REIT returns ($\lambda$)
MAPE Statistics - all types.
MAPE Statistics - Apartments.
December 2020 forecast and the GFC.
### MAPE Statistics - Apartments.

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>mape</th>
<th>sd</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Baseline</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>forecast</td>
<td>-0.00175</td>
<td>0.00555</td>
<td>0.00663</td>
<td>-0.01827</td>
<td>0.01106</td>
</tr>
<tr>
<td>final period</td>
<td>-0.00085</td>
<td>0.00295</td>
<td>0.00355</td>
<td>-0.00862</td>
<td>0.00740</td>
</tr>
<tr>
<td>all</td>
<td>-0.00003</td>
<td>0.00009</td>
<td>0.00010</td>
<td>-0.00029</td>
<td>0.00019</td>
</tr>
<tr>
<td><strong>MKF repeat sales model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>forecast</td>
<td>-0.00047</td>
<td>0.00361</td>
<td>0.00446</td>
<td>-0.01348</td>
<td>0.00821</td>
</tr>
<tr>
<td>final period</td>
<td>-0.00026</td>
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<tr>
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<td>0.00009</td>
<td>-0.00019</td>
<td>0.00018</td>
</tr>
<tr>
<td><strong>MKF repeat sales model - robust</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>forecast</td>
<td>-0.00061</td>
<td>0.00351</td>
<td>0.00424</td>
<td>-0.01205</td>
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<td>final period</td>
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<td>-0.00017</td>
<td>0.00014</td>
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December 2020 forecast and the GFC
Introduction
Methodology
Data
Results - Estimates on $\lambda$
Results - Indexes
Fit
December 2020 forecast and the GFC

Forecast
September 2007
October 2007
November 2007